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Numerical modelling of free-surface flows in ship hydrodynamics

U. P. Bulgarelli*,†, C. Lugni and M. Landrini

INSEAN, The Italian Ship Model Basin, Via di Vallerano 139, 00128 Roma, Italy

SUMMARY

Current trends in the investigation of ship hydrodynamics are reviewed with emphasis on the problem of wave-body interaction. This includes the classical seakeeping problem and as a special case, the problem of prediction for the drag encountered by a ship advancing in calm water. Specific issues related to the numerical treatment of the air–water interface are examined, with emphasis on the modelling of wave breaking. The discussion on the large-scale modelling of the flow around ships is focused on the prediction of wave loads, ship motions and resistance in calm water. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: free surface; ship hydrodynamics; resistance; seakeeping

1. INTRODUCTION

We review some research trends in the numerical modelling of the flow about a surface ship moving either in calm water or through waves. Results by a variety of approaches are presented to show present limits and potentiality of the considered methods.

Generally speaking, the flow around a ship sailing through waves is described by the conservation of mass and momentum which ultimately lead to the Navier–Stokes equations. The boundary domain is formed by solid (possibly moving) boundaries of various kinds, over which a no-slip condition applies, and by the air–water interface $\mathscr{F}$. Here, a kinematic constraint arises due to the physical condition that there is no mass flux across $\mathscr{F}$ which, indeed, moves as a material surface. The kinematic condition can be stated both in the Eulerian and in the Lagrangian form according to whether the interface between air and water is single valued or not. In the case that the fluid above the water is a dynamically non-active fluid, the classical free-surface scheme is obtained, and the dynamic boundary condition requires that the stress at the interface surface balances the ambient pressure and the surface tension

*Correspondence to: U. P. Bulgarelli, INSEAN, Via di Vallerano 139, 00128 Roma, Italy.
†E-mail: up.bulgarelli@insean.it

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restoring force. In particular tangential stress components must vanish. If the fluid domain is infinite in extent, suitable radiation conditions have to be specified and, for an initial boundary value problem, the flow variables have to be fixed at a reference time.

A variety of simplifying assumptions can then be made to alleviate the complexity of the above recalled mathematical problem. In particular, when dealing with free-surface flows, a powerful simplification consists in assuming small perturbations of the free surface, leading to linear (or weakly non-linear) models. This hypothesis may or may not be combined with the assumption of negligible viscous effects, which then leads to potential-flow models.

In the following, algorithms are first presented to deal with (highly) non-linear free-surface flows. In practice, this limits the application to two-dimensional problems, though three-dimensional solutions are in principle feasible. In the second part, free-surface breaking is ruled out and three-dimensional results for practical geometries are presented.

2. MODELLING OF FREE-SURFACE FLOWS WITH WAVE BREAKING

Great advances have taken place in the simulation of water wave propagation by boundary element methods through the formation of a plunging breaker, both in deep and in shallow water. However, it has been apparent for some time that new numerical approaches are necessary if we are to gain understanding and to simulate processes in water waves after the jet splashes down throughout the post-breaking phase in the case of deep water waves, and continually through the cyclical splashing phase characteristic of bores, for instance.

The existence of an appropriate numerical approach would immediately surmount the problems which theories have long had to try to predict such as the behaviour of highly non-linear flows like breaking waves on beaches, or high-amplitude sloshing in containers, or waves around ships, using highly idealized mathematical methods. In fact, it is only after seeing the enormously complicated behaviour of the free surface in producing ricochets, backward facing jets, folded shapes, holes, and vortical structures, that a true appreciation of the hydrodynamic complexities can be realized.

2.1. Methods

Plenty of numerical methods to deal with non-linear free-surface flows have been developed. They can be roughly divided into:

- Boundary-discretization methods, relying on the integral formulation of the boundary value problem. They are mostly used for inviscid-irrotational flows, which are the only ones considered in Section 2.1.1.
- Volume-discretization methods, in principle applicable to both viscous and inviscid problems, based on the pointwise application of the field equations.

Several reviews on numerical methods for free-surface flows exist. Here just details concerning marine related hydrodynamics are reviewed.

2.1.1. Boundary discretization methods. Regardless of the actual integral formulation adopted (Green second identity, (mixed) indirect layer ansatz), the numerical procedure can be roughly described by the following two-step procedure:

1. For a fixed time the velocity field (mostly described in terms of a velocity potential) has to be evaluated. The boundary conditions are the normal velocity component on solid boundaries and the potential (or its tangential derivatives) on the free surface.
2. Free-surface boundary conditions and equations of body motion are integrated in time to update the geometry of the boundary domain and the relevant boundary data. This allows for repeating step 1.

In this framework large-scale computations are made feasible both by increasing the accuracy of the boundary-integral equation solver and by accelerating the solution of the integral equations. The first point is accomplished either by adopting high-order boundary-element methods, or by using desingularized methods. The second issue can be accomplished by each of the following techniques: domain-subdivision techniques, clustering techniques, multipoles expansion with fast summation coupled to iterative solvers. Apparently, the latter is the most exploited in the naval context (low-order BEM [1], desingularized [2], spectral accurate but only two dimensional [3]).

Neglecting fundamental studies concerning the wave dynamics, the main goal of the above approaches is the global prediction of the inviscid part of the free-surface induced loads. Finer studies are devoted to detect breaking events (relevant to reduce energy loss and remote sensing) [4] and to predict water on deck occurrence [5]. In the field of seakeeping of ships, after coupling with the equations of body motion, the goal is the prediction of the non-linear ship dynamics.

As a matter of fact boundary-integral approaches appear to be able to detect and follow wave breaking up to the formation of jets or plunging breakers. Unfortunately, the post-breaking behaviour still lacks theoretical and numerical descriptions.

2.1.2. Volume discretization methods. Because of the high-Reynolds numbers typically encountered in ship hydrodynamics, viscous effects are practically relevant only in proximity of the boundary domain and, in principle, volume discretization methods allow one to properly handle the boundary layers at the solid boundaries and at the free surface. Unfortunately the practical implementation of the viscous free-surface boundary conditions is not straightforward. Difficulties related to the unknown free-surface position can be resolved by considering the transient problem, starting from rest, even in steady-flow computations. The hull is thus accelerated to the requested Froude number and the time integration is continued until steady-state conditions have been achieved. (This procedure corresponds to usual practice in towing tank experiments—with all the caveats.) The kinematic boundary condition is used to update the free-surface position in each step. Difficulties arise at the hull where fluid cannot move tangentially due to the no-slip condition. So usually the vertical position of the free surface at the first point off the hull is used also on the hull itself. Park and Miyata [6] computed for the first time breaking bow waves for a tanker model obtaining good agreement with experiments, but few research groups have managed to follow suit so far. Simplified viscous-flow computations for ships still assume the free surface to be a symmetry plane.

When the viscous behaviour of the free surface is considered, most of the steady and unsteady viscous methods for ship flows simplify the dynamic free-surface boundary condition: tangential stresses are usually unconstrained and often the pressure is simply required to be atmospheric. Eventually the inviscid free-surface boundary conditions are used in conjunction with most of the Navier–Stokes solvers, an approximation valid for weak free-surface boundary layer and relatively high Reynolds numbers. The problem of turbulence models (and their specific boundary conditions) near the free surface has not been addressed in ship flows and generally the same conditions as for symmetry planes are used.

Further difficulties are related to ship changes of trim and sinkage (squat) due to the wave effects. In principle, coupling with the equations of body motion is required. The squat is usually neglected even in research applications, with References [7, 8], being notable exceptions.

Besides the interest for solving viscous flows, volume discretizations methods are of interest because they are apparently more robust than boundary discretization in dealing with breaking flows. A number of methods try to capture wave making with various degrees of success. Eventually, only methods capable of modelling that include breaking waves should survive. Some research groups (e.g. Miyata at the Tokyo University, Larsson at the Chalmers University) have spearheaded the development with sometimes spectacular applications, but many problems remain to be solved world wide.

Different methods to determine the shape of the free surface can be classified into two groups:

1. *Boundary-fitted methods* or interface-tracking methods, which define the free surface as a sharp interface whose motion is followed. The physical domain is then mapped into a more regular computational domain on which the problem is solved via finite differences [9], or finite volume techniques [10, 11]. Problems are encountered when the free-surface starts folding or when the grid has to be moved along the walls of a complicated shape (like a real ship hull geometry). A major problem is, in fact, mapping of the physical domain in the computational ones. Unstructured and multi-block grids can represent the final solution to deal with complex geometries and when large free surface deformations are involved.

2. *Eulerian-grid methods* or interface-capturing methods, in which computation is performed on a fixed grid (usually rectangular) which extends also over the air region. The free surface is not defined as a sharp interface and its shape is determined by finding the cells which are only partially filled with water. Typical schemes are

   - Marker-and-cell (MAC) scheme. Some massless particles (markers) are introduced into the water near the free surface at the beginning and followed during the calculation. The scheme can compute complex phenomena like wave breaking, e.g. Reference [6]. However, the computing effort is large since in addition to solving the equations governing the fluid flow, one has to solve the equations describing the movement of a large number of particles.
   - Volume-of-fluid (VOF) scheme. The transport equation for the void fraction of the water (1 for completely filled, zero for completely empty cells) is solved (in addition to the usual conservation equations of mass and momentum). The method is more efficient than MAC and can also be applied to breaking waves. However, the free-surface contour is not sharply defined and special techniques had to be developed to obtain an accurate profile with reasonable numbers of cells, e.g. References [12–15].
   - Level-set technique. In this interface capturing technique a distance function ϕ is defined at least in a portion of the computational domain close to the interface. Upon considering the level set function as an additional passive scalar convected by

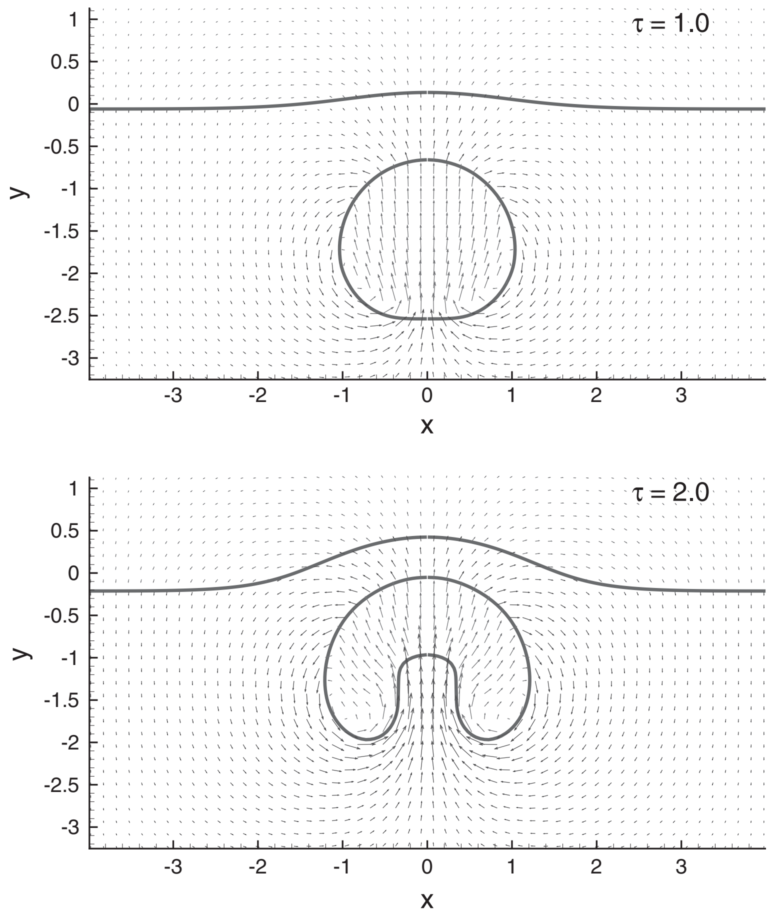

Figure 1. Evolution of a air bubble rising against a free surface. The numerical solution is obtained by solving the Navier–Stokes equations, both in air and water. The interface between the two fluids is reconstructed by a level set algorithm.

the fluid, the interface is found by looking at that subset of values [16]. The interface is no longer a sharp interface but the subset of the level set function has to remain relatively small to avoid a poor description of the waves.

An example of the application of this technique is given in Figure 1, where the flow generated by an air bubble rising against a free surface is depicted (Figure 2). Figure 3 shows the interpolation procedure to capture the interface.

The major advantages of such methods are robustness, relative simplicity and the ability to handle complex geometry and complex events (breaking and bubble formation). On the other hand, Eulerian-grid methods are often of low order and lack accuracy when surface tension or viscous boundary layer phenomena are dominant.

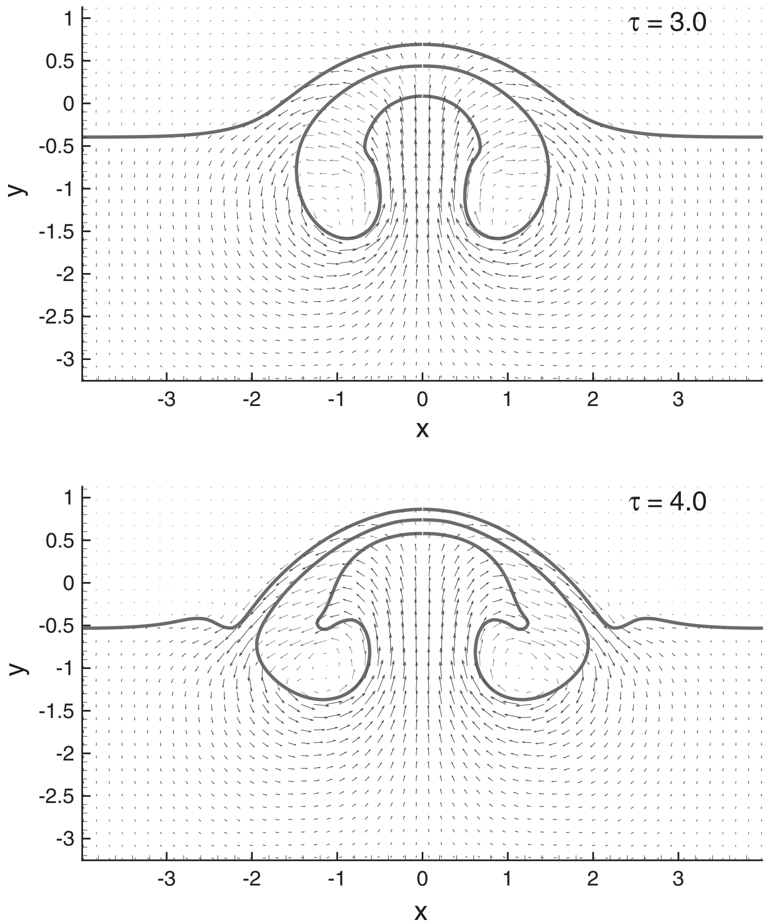

Figure 2. For caption see Figure 1.

3. *Gridless methods*: In principle, the problem of adapting the grid to large deformations of the fluid domain can be approached by removing the grid. This is the case of gridless methods, in which the field equations are discretized by using points irregularly scattered over the computational domain [17]. In some cases, the computational points have a physical meaning and represent real fluid particles. In this case a Lagrangian method is obtained, and was first introduced by Monhagan [18], and called smooth particle hydrodynamics (SPH). In all the cases, the key concept is the possibility of representing fluid quantities in term of interpolating operators based on scattered data.

   A modified SPH method has been recently applied to ship flows with wave breaking by Tulin and Landrini [19] and Landrini *et al.* [20]. The particle method is applied in combination with a slender body theory for the flow around a frigate. The idea is that longitudinal gradients of relevant flow quantities are small compared

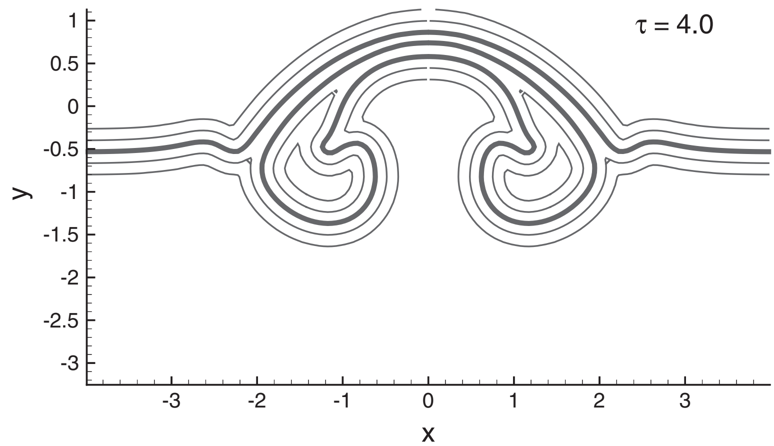

Figure 3. Level-set reconstruction of the interface between two fluids. Contour lines of the distance function ϕ. The actual location of the interface is obtained by interpolating the $\phi = 0$ line (thick line).

with vertical and transverse gradients. On those grounds, calculations are carried out in two dimensions, vertical and transverse to the ship, and successively in time. Figure 4 shows the evolution of the upper layer of free-surface particles. The collapse of the bow wave is apparent: the free-surface flow is not much decelerated before the stern, but upon reaching it, is deviated sharply upwards, rises on and eventually levels off and falls down. An entire thin sheet is formed in this process and appears as a splash on the side of the hull. The relaxation of these splashes is the prime source of divergent waves. In the present case, the strong flare causes the bow splash to break at the hull. Striking features are the splashes, formation of cavities entrapping air and the creation of vortical structures. Figure 5 shows the three-dimensional steady wave pattern around the hull, reconstructed by collecting successive free-surface configurations.

3. MODELLING OF NON-BREAKING SHIP FLOWS

In spite of the physical and practical importance of wave breaking, the description on larger spatial scales of the fully three-dimensional flow around a ship forces us to simplify the physical problem.

3.1. Resistance

Wave breaking is usually neglected when solving the steady flow generated by a ship advancing in calm water. At present, methods to solve the Reynolds averaged Navier–Stokes equations are available and successful results have been achieved up to the order of 10^7 Reynolds number, which is the typical Reynolds number for models in a towing tank. The

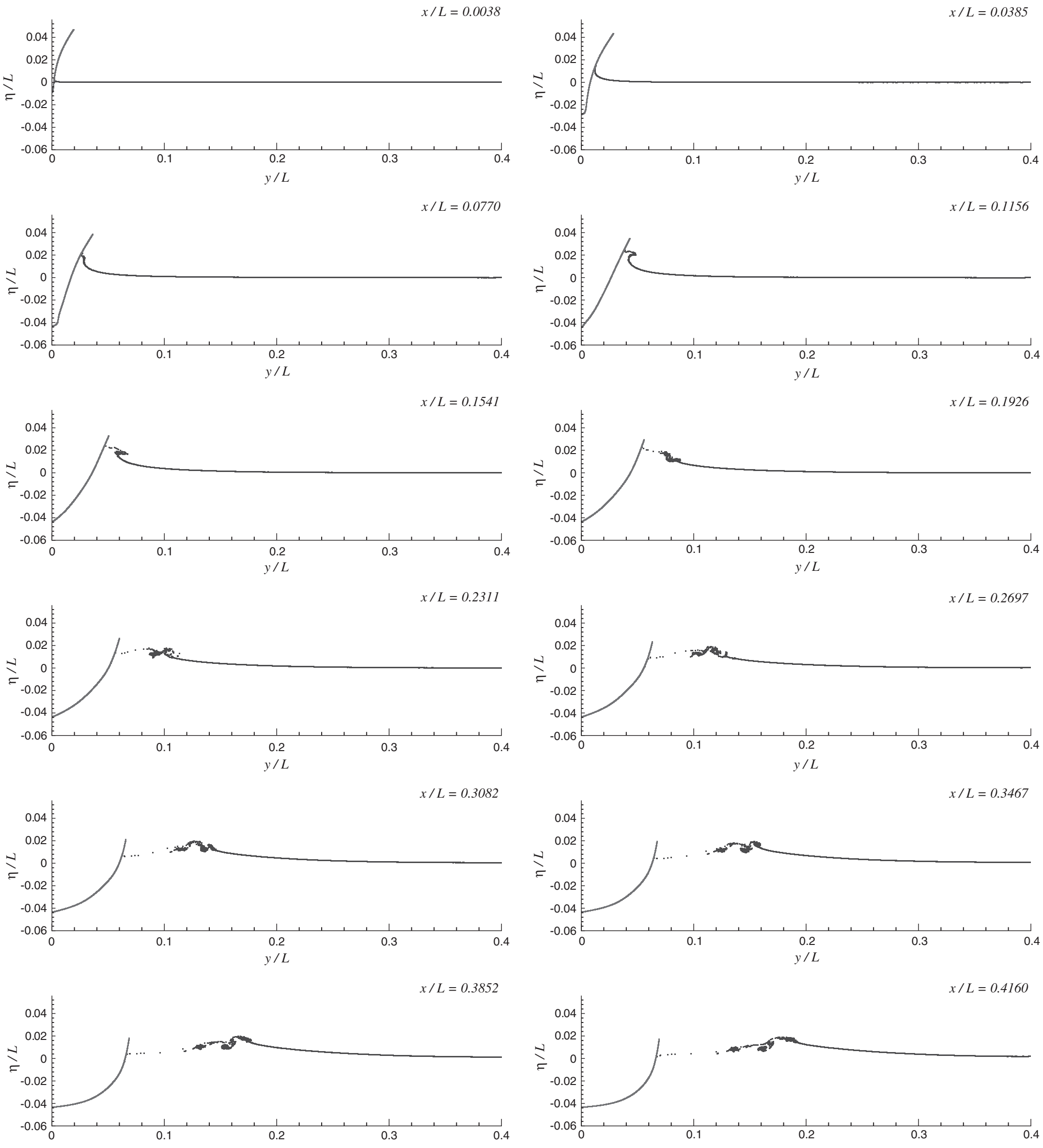

Figure 4. Slender body computations of the bow wave generated by a ship in forward speed. The Euler equations are solved by a modified SPH method. The time increases from top down, and from left to right.

next step to reach full-scale Reynolds numbers, $\mathcal{O}(10^9)$, still requires an increase in computer capabilities and in numerical models. A full example of the present capabilities reached by RANSE solvers for computing the flow around a ship running in calm water is given in Reference [21].

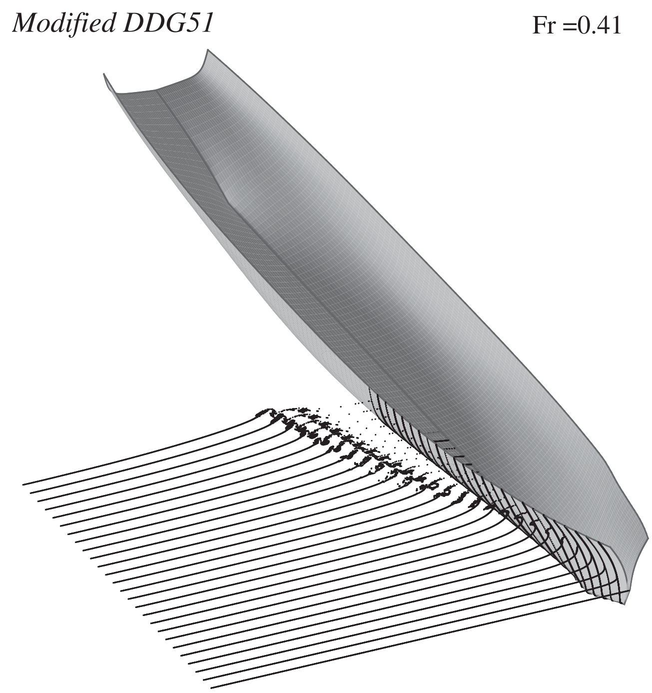

Figure 5. Bow wave pattern around a frigate by a slender body theory combined with a modified SPH method.

Here we show some state-of-the-art RANSE computations for a ship in steady drift, i.e. with an angle of attack.

The solution of the free-surface turbulent steady flow past a ship hull is obtained as the asymptotic solution of the unsteady pseudo-compressible Navier–Stokes equation [22]. The mathematical equations are approximated by a discrete finite volume scheme; the problem is solved on a domain whose boundaries are moved to fit the actual free surface.

The viscous term in the equations are approximated by centred differences, whereas velocity and pressure at the interface in the inviscid Eulerian fluxes are evaluated by a second-order ENO-type scheme [23]. Runge–Kutta scheme is used to update the numerical solution. Convergence rate to steady state is enhanced by means of a full approximation scheme—full multigrid (FAS-FMG) algorithm [24].

In the simulations, three different turbulent models have been employed: the Baldwin-Lomax algebraic turbulent model [25], the Spalart-Allmaras one equation model [26] and the Chang-Hsieh-Chen $k-\varepsilon$ model [27].

The comparison of measured [28] and computed [29] data for a ship hull (Series 60 $C_{\mathrm{B}}=0.6$) in steady drift motion are shown in Figure 6. In particular the contour wave pattern for $Fr=0.316$ and for two different drift angles are considered. The overall agreement is rather good, as confirmed by the wave profiles along the hull (Figure 7). In this case the measured data (symbols) are compared with the numerical results obtained by refining the numerical grid.

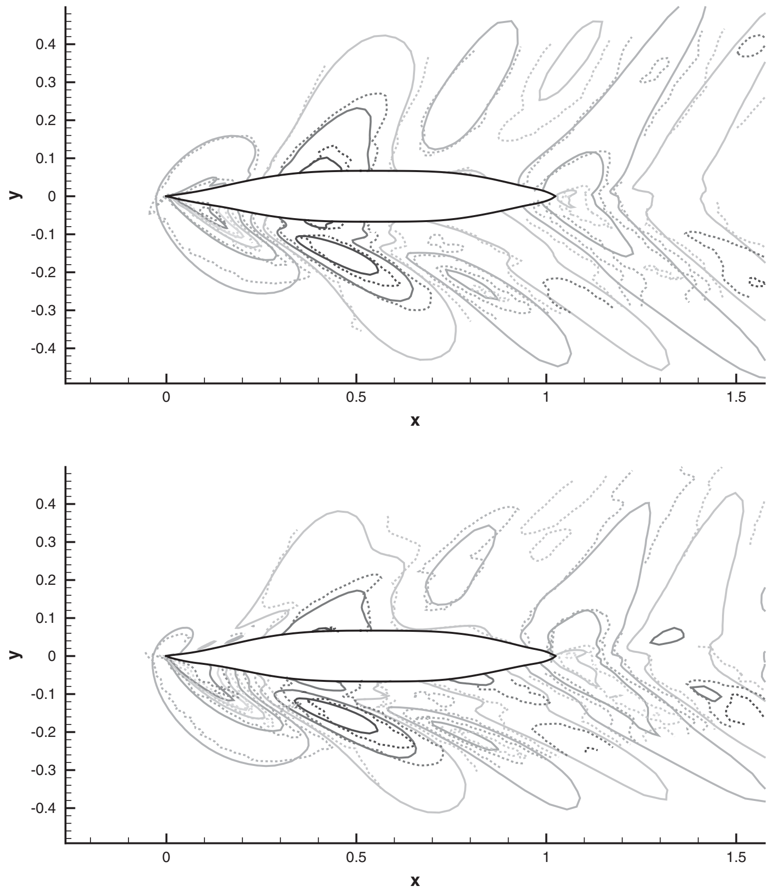

Figure 6. Contour of wave pattern around a Series 60 $C_b = 0.6$ in steady drift motion, $Fr = 0.316$. Top: 5°, Bottom: 10°. Numerical results (solid lines) are compared with the experimental data by Longo (dashed lines).

3.2. Seakeeping

Reliable tools to predict wave-induced motions usually rely upon the hypotheses of potential flow and small amplitudes of free-surface deformations and body motions.

On these grounds, linearized frequency domain strip theories [30] are largely employed because of the good compromise between simplicity, accuracy and computer requirements. Fully three-dimensional algorithms in the frequency domain are also available [31], but in spite of greater accuracy, it is more limited in its ability to deal with arbitrary geometries (e.g. multi-hulls) and following waves.

On the other hand, the fully non-linear three-dimensional simulation in time domain is the natural avenue to deal with extreme wave conditions. For ship motions, the fully non-linear approach is pioneered by the group led by R. Beck at University of Michigan through a desingularized Rankine panel method [32]. The (weakly) non-linear code SWAN2

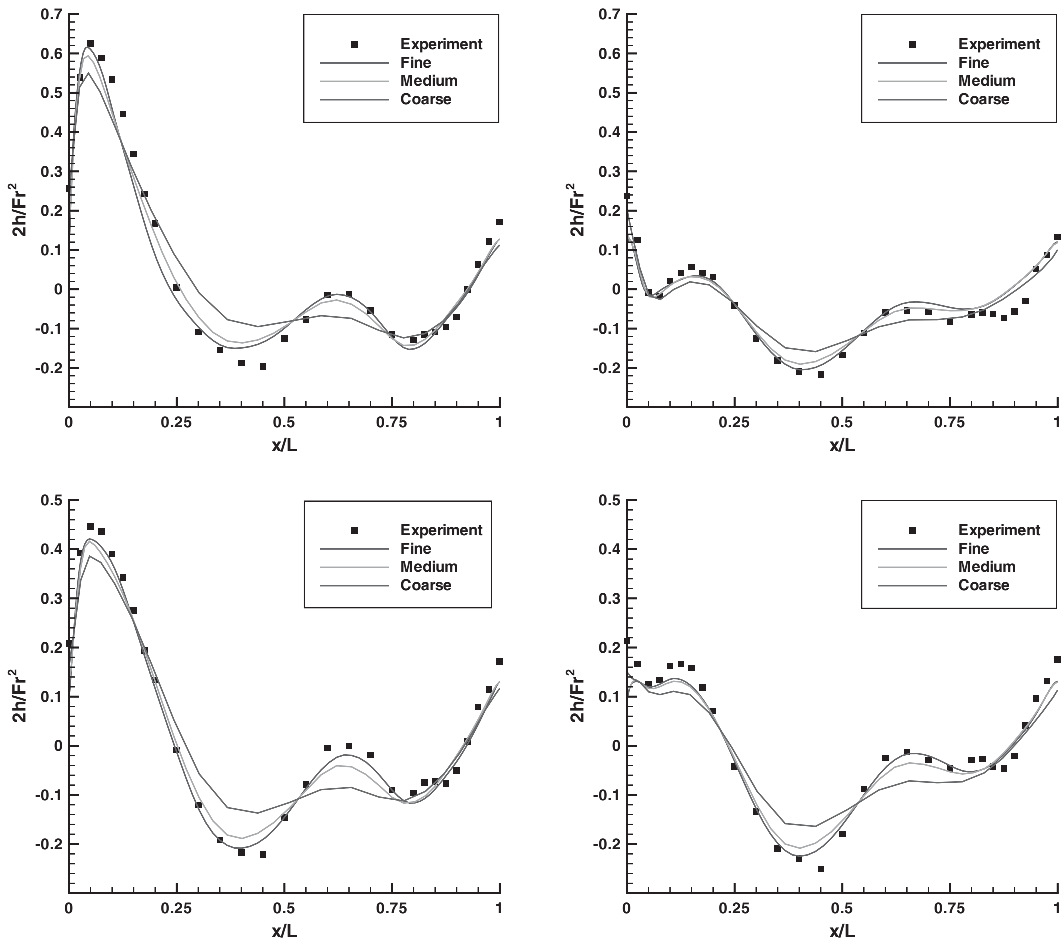

Figure 7. Wave profiles along a Series 60 $C_b = 0.6$ hull in steady drift motion, $Fr = 0.316$ for $\alpha = 5^\circ$ (top) and $\alpha = 10^\circ$. Left: pressure side. Right: suction side. The numerical results for three different grids are compared with the experiments by Longo (symbols).

developed at MIT [33] is the most extensively tested time domain code documented in literature.

The following results represent the typical capabilities of state-of-the-art potential flow codes for the advanced analysis of the seakeeping of ships.

The solution of the unsteady motion of a ship in a seaway is obtained by an inviscid model. In the linear framework, the case of small oscillations of the hull with respect to the mean trim and sinkage attained by the ship when advancing in calm sea with constant forward velocity in the x-direction, is considered. The perturbation velocity potential satisfies the Laplace equation with the no-penetration boundary condition on the mean wetted hull and with the linearized kinematic and dynamic free-surface conditions on the mean free-surface

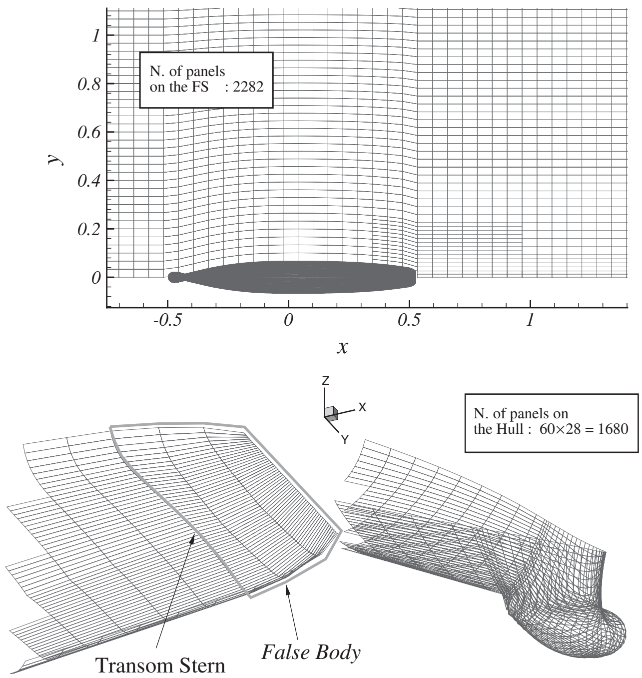

Figure 8. Seakeeping simulation for a frigate-type ship. Top: typical discretization of the free surface. Bottom: details the discretization of transom and bow regions.

level. The solution of the fluid dynamic problem, coupled with the body motion through the hydrodynamic loads, is obtained by solving a kinematic and an evolution problem. A mixed Neumann–Dirichlet boundary value problem for the Laplace equation can then be stated and the layer ansatz is assumed to represent the solution. A set of integral equations is deduced by collocating on the free surface and on the body, and the numerical solution is achieved by a standard low-order panel method [34].

Once the velocity potential is computed on the boundary domain, the free-surface equations, as well as the equations of the body motion, are stepped forward by a fourth-order Runge–Kutta scheme.

In Figure 8 a typical computational domain for a frigate-type ship (Model 5415 of the David Taylor Model Basin), characterized by a transom stern and a sonar dome, is shown. An apparent feature of the adopted discretization is the use of a non-structured grid for the free surface. This is a key point to easily deal with multi-hull and transom stern and also to allow an easy local mesh refinement, if required.

The motions of the frigate hull in seaway are shown in Figure 9 relatively at $Fr = 0.28$ (top figure) and $Fr = 0.41$ (bottom figure). In particular the response amplitude operators for heave and pitch motions are obtained by interacting the hull with a wave packet [35]. At the initial time the hull advances in calm sea with constant speed, while on the opposite end of the fluid domain the wave train is generated. Around the focusing point the wave packet interacts with the model which is free to respond in surge, heave and pitch. The interaction

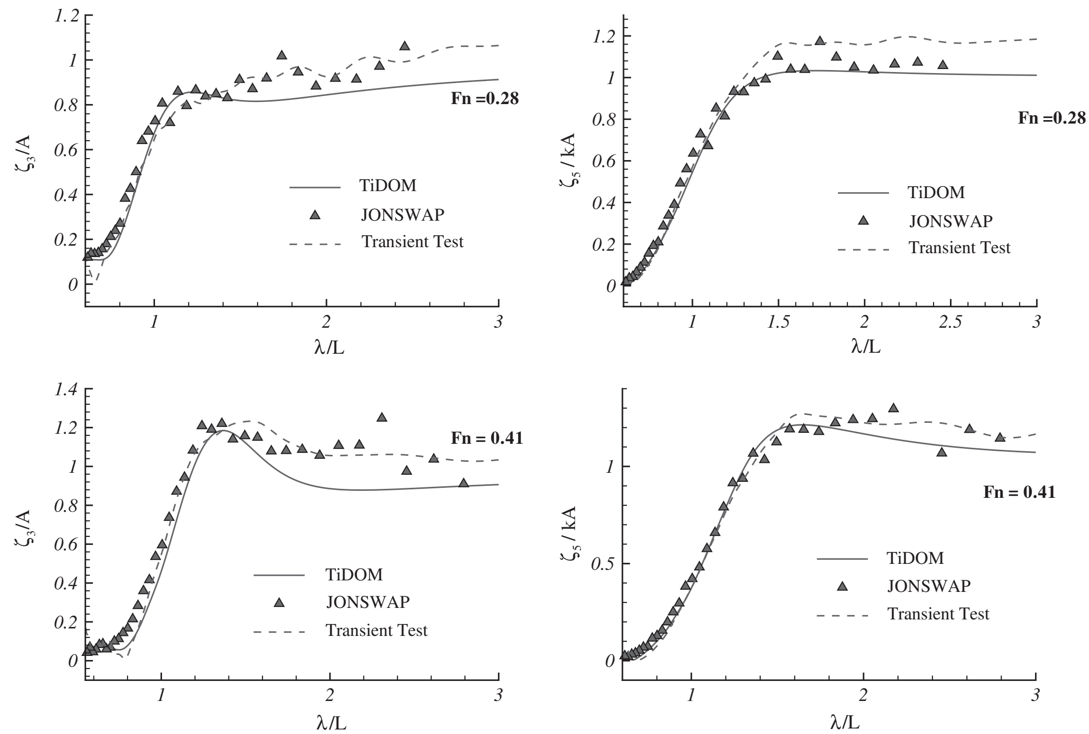

Figure 9. Heave (left column) and pitch (right column) Response Amplitude Operator for a frigate-type hull advancing in seaway at $Fr = 0.28$ (top figure) and $Fr = 0.41$ (bottom figure). The numerical results (solid line) are compared with the experimental data obtained by a transient test technique (dashed line) and by a traditional test in irregular wave (symbol).

time is rather short, and the experiment ends with the hull running in quasi-calm water. In principle, one test suffices to evaluate the response within the significant frequency range. The short duration of the test and the small wave elevation downstream from the focusing point avoids problems due to reflected long waves.

The results of this recent technique applied both experimentally and numerically [36], are compared (Figure 9) with the measured data obtained by applying a traditional test in irregular waves [37]. On the whole, a satisfactory agreement is obtained.

The wave pattern radiated by a blunter ship (a Series 60 hull with block coefficient 0.8) is analysed in Figure 10. The model is forced to oscillate sinusoidally in heave and pitch, while advancing with constant speed at $Fr = 0.2$. The radiated waves are measured by an array of transducers located longitudinally at a distance $2y/L = 0.2038$ from the centre plane of the ship. The signal recorded in time at each location is then Fourier transformed, and the resulting real and imaginary parts are plotted in Figure 10, together with the numerical results (see Figure 11) [38]. We used solid and dashed lines for the experimental real and imaginary parts, and $\triangle$ and ∇ for the corresponding numerical data. The latter are obtained by following the same procedure as in the experiments. The oscillation frequency $\hat{\omega}$ de-

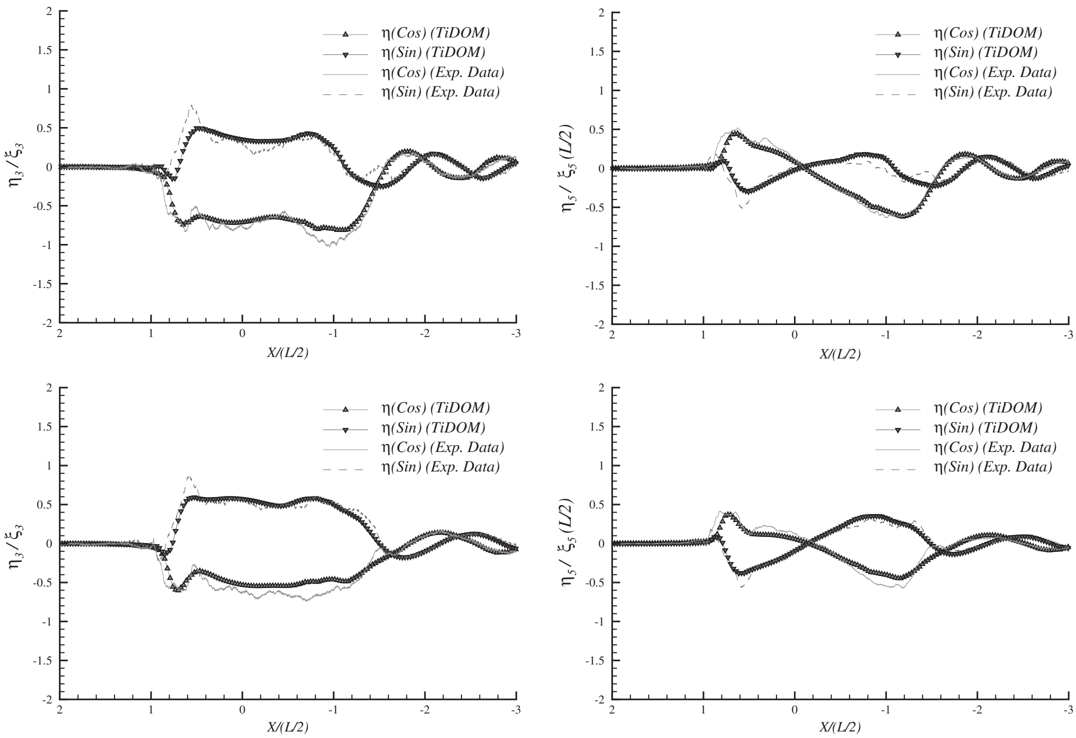

Figure 10. S60, $C_b = 0.8$ (tanker): longitudinal cuts of the wave system generated by forced heave (left) and pitch (right) motion. The oscillation frequency increases from top down and lengths are divided by half of the ship length L. The hull is located between -1 and 1 and is advancing from right to left.

creases from top down. Specifically, $L/\hat{\lambda} = 4.615, 3.183, 1.91, 1.11$, where the linear dispersion relation $2\pi/\hat{\lambda} = \hat{\omega}^2/g$ is used to define the reference wavelength $\hat{\lambda}$. The overall agreement is rather good, although we notice the tendency to smooth out the peaks at the bow. Probably, this indicates the need for a finer discretization in the bow region. In spite of this, by using the same grid, the predicted RAOs (not shown) are in excellent agreement with the experiments.

4. CONCLUDING REMARKS

Current trends in the simulation of free-surface flows around surface ships are reviewed with emphasis on the prediction of wave pattern and loads.

Results are generally promising with main flow features qualitatively captured, even when large non-linearities appear. Achieving good quantitative agreement requires further effort in the modelling of breaking waves around ships (possibly with two-phase flow effects) and advances in the modelling of turbulence.

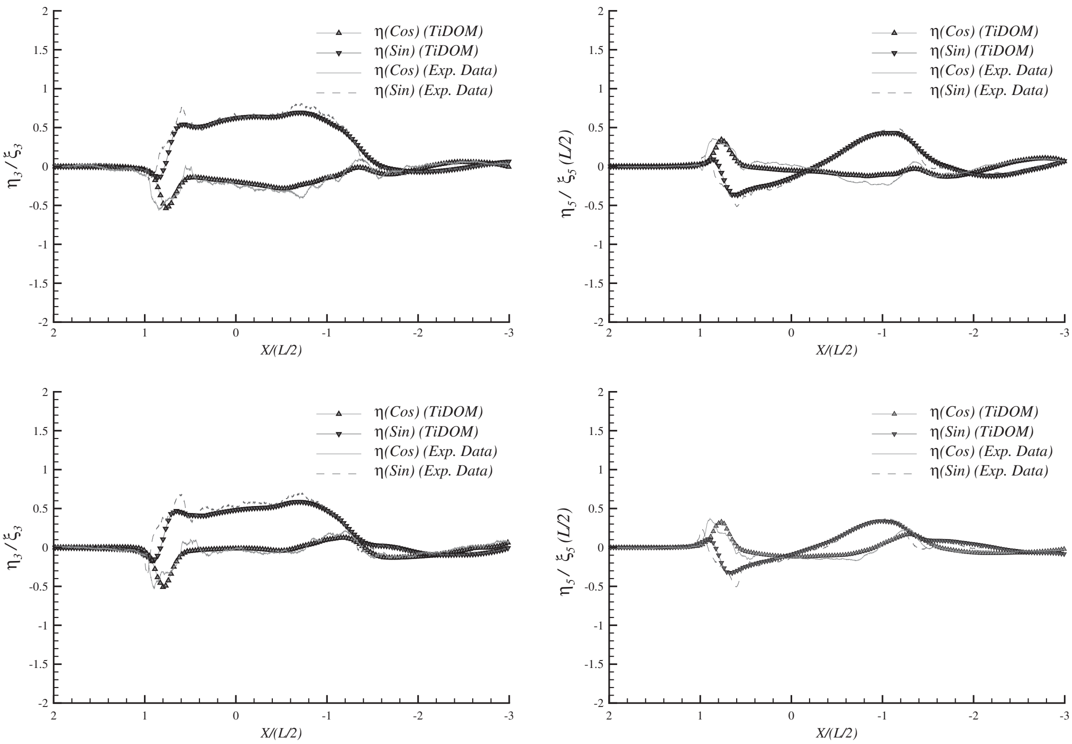

Figure 11. For caption see Figure 10.

Complementary experimental fluid dynamics, in the spirit of the Gothemburg 2000 Workshop [21], will play a crucial role for future developments. In particular, moving towards the prediction of the unsteady motion of ships (maneuverability) requires the availability of time-accurate global and local experimental data for code validation.

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